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graphs that had been omitted. Or the course can be made a lecture course with considerable work in assigned problems and outside reading. Reye's *Geometry of Position* (volume 1, English translation by T. F. Holgate) is out of print, but it is likely to be in the library so that it can be used for reference. The *Projective Geometry* by Veblen and Young is too advanced and abstract for students who have had only two years of college mathematics, and besides it is not suitable for a short course. But it can be used to advantage for reference at times. Emch's *Projective Geometry* and Miss Scott's *Modern Geometry* get at the subject mostly from the side of analytic geometry, but they can be used for reference at times. There are some English books that the teacher may want to have in the library for occasional reference, although no one of them is suitable for use as a textbook for the course, namely, Russell's *Elementary Treatise on Pure Geometry*, Milne's *Elementary Treatise on Cross Ratio Geometry*, Filon's *Projective Geometry*, Pickford's *Elements of Projective Geometry*, and Durell's *Plane Geometry for Advanced Students.*

It is to be hoped that some one will publish a book on synthetic projective geometry that will be suitable for use as a textbook for an introductory course in the subject. In the meantime the course if given will have to be conducted in one of the ways suggested above.

A NOTE ON THE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS.

By C. R. MAC INNES, Princeton University.

In showing how to solve linear differential equations with constant coefficients, the usual way of finding the complementary function seems to be to guess at the various parts and then to prove that the guesses are correct. A much more satisfactory way is to define the differential operators $D-\alpha$, $D-\beta$, etc., to show that they obey the laws of addition and multiplication, and then to proceed in a straightforward way to solve the problem.

Having shown that $\frac{d^2y}{dx^2}-(\alpha+\beta)\frac{dy}{dx}+\alpha\beta y=0$ may be written as $(D-\alpha)(D-\beta)y=0$, one sees that a solution of $(D-\beta)y=0$ is a solution of the original equation since $(D-\alpha)0\equiv 0$. Since the factors are commutative any one may come last and we can get all the parts of the complementary function except those arising from repeated factors.

If $(D-\alpha)(D-\beta)y=X$, put $(D-\beta)y=v$; then $(D-\alpha)v=X$. Integrating this in the usual way one gets

$$v=(D-\beta)y=e^{\alpha x}\int e^{-\alpha x}X\,dx.$$

Hence the rule: *To remove a factor* $(D-\alpha)$ *from the left side, multiply the right member by* $e^{-\alpha x}dx$*, integrate and multiply by* $e^{\alpha x}$.

If we have $(D-\alpha)^2y = 0$, we may therefore get $(D-\alpha)y = c_1e^{ax}$. On repeating this operation,

$$y = e^{ax}\int e^{-ax}c_1e^{ax}dx = c_1xe^{ax} + c_0e^{ax}.$$

If we have $(D-\alpha)^3y = 0$, we get in precisely the same way

$$y = c_2x^2e^{ax} + c_1xe^{ax} + c_0e^{ax}.$$

The complementary function would then be completely known on proving the usual theorem that if $v_1, v_2, \cdots v_n$ are solutions of $f(D)y = 0$ then $c_1v_1 + c_2v_2 + \cdots c_nv_n$ is also a solution.

One need hardly draw attention to the fact that the complete value of y will be obtained if the factors of

$$(D-\alpha_1)(D-\alpha_2)\cdots(D-\alpha_n)y = X$$

are removed in turn by the above rule, the constant of integration being introduced each time.

A GRAPHICAL SOLUTION OF THE DIFFERENTIAL EQUATION OF THE FIRST ORDER.*

By T. R. RUNNING, University of Michigan.

The following graphical solution of the differential equation of the first order was derived while trying to solve a problem involving two such equations. As an illustration, these equations will be solved after a discussion of the method of solution has been given.

If the primitive of a differential equation which expresses the condition of a problem can not be found, or can be found with difficulty, it may be desirable to have a method of approximate solution which does not involve a great deal of labor. It is thought that the method given is more easily carried out than any other which has come to the writer's attention.[1]

The solution depends upon the simple fact that the area under the derived curve is equal to the corresponding ordinate of the integral curve. In Fig. 1

$$\text{Area } OPQ = RQ.$$

If in the equation

$$\frac{dy}{dx} = f(x, y)$$

we assign particular values to y and plot the resulting equations with dy/dx as

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* First part of a paper presented to the American Mathematical Society at Madison, September 9, 1913.

[1] For a comparison of approximate methods see "Beitrag zur näherungsweisen Integration totaler Differentialgleichungen," by W. Kutta, *Zeitschrift für Mathematik und Physik*, Vol. 46, pp. 435–453.